Superfield approach to BRST cohomology

Superfield approach to BRST cohomology

R P Malik

S N Bose National Centre for Basic Sciences, Block-JD, Sector-III, Salt Lake,
Calcutta 700 098, India

E-mail: malik@boson.bose.res.in

Abstract
In the framework of superfield formalism, we discuss some aspects of the
cohomological features of a two (1+1)-dimensional free Abelian gauge theory
described by a Becchi–Rouet–Stora–Tyutin (BRST) invariant Lagrangian
density. We demonstrate that the conserved and nilpotent (anti-)BRST and
(anti-)co-BRST charges are the generators of translations along the
Grassmannian directions of the four (2+2)-dimensional supermanifold. A
bosonic symmetry is shown to be generated by a Noether conserved charge that
generates a translation along a bosonic direction of the supermanifold which
turns out to be equivalent to a couple of successive translations along the two
different and independent Grassmannian directions of the same supermanifold.
Algebraically, these charges are found to be analogous to the de Rham
cohomology operators of differential geometry.

PACS numbers: 11.10.−z, 02.40.−k, 11.15.−q, 11.30.−j

1. Introduction

The existence of some new local, covariant and continuous symmetries and their possible
connections with the mathematics of differential geometry, cohomology and Hodge
decomposition theorem (HDT) for the two (1+1)-dimensional (2D) free as well as interacting
(non-)Abelian gauge theories have been recently established in a set of papers [1–5]. The
topological nature of the 2D free Abelian and self-interacting non-Abelian gauge theories
(having no interaction with matter fields) has also been demonstrated by exploiting the
conserved and nilpotent (anti-)BRST charges ($Q_{(a)b}$), (anti-)dual BRST charges ($Q_{(a)d}$), a
conserved ghost charge Q_g and a bosonic charge $Q_w = \{Q_d, Q_b\} = \{Q_{ab}, Q_{ad}\}$ which
generate some interesting local, covariant and continuous symmetry transformations for the
2D BRST invariant Lagrangian density [6]. These local conserved charges have also been
utilized to express HDT in the quantum Hilbert space of states where any arbitrary state $|\Psi\rangle_n$
with the ghost number n, (i.e. i$Q_g|\Psi\rangle_n = n|\Psi\rangle_n$) can be written as the sum of a harmonic state

$|\omega\rangle_n$ ($Q_w|\omega\rangle_n = 0$, $Q_b|\omega\rangle_n = 0$, $Q_d|\omega\rangle_n = 0$), a BRST exact state $Q_b|\xi\rangle_{n-1}$ and a co-BRST exact state $Q_d|\chi\rangle_{n+1}$ as[1]

$$|\Psi\rangle_n = |\omega\rangle_n + Q_b|\xi\rangle_{n-1} + Q_d|\chi\rangle_{n+1} \equiv |\omega\rangle_n + Q_{ad}|\xi\rangle_{n-1} + Q_{ab}|\chi\rangle_{n+1}. \tag{1.1}$$

The above equation encodes the fact that, in the BRST formalism, there are two sets of conserved charges (namely (Q_b, Q_d, Q_w) and (Q_{ad}, Q_{ab}, Q_w)) that are analogous to the de Rham cohomology operators of differential geometry. The origin for the existence of this mapping: $(Q_b, Q_{ad}) \Leftrightarrow \mathrm{d}$, $(Q_d, Q_{ab}) \Leftrightarrow \delta$, $Q_w = \{Q_{(a)b}, Q_{(a)d}\} \Leftrightarrow \Delta$ has not been made clear in our earlier works [1–6]. *In the present paper, we clarify the existence of such a mapping*.

In the physical four (3+1)-dimensional (4D) flat Minkowskian spacetime, the above type of symmetries and corresponding local and conserved charges have been exploited for the definition of the celebrated HDT in the context of free Abelian two-form gauge theory [11]. Furthermore, these local and conserved charges have been shown to obey exactly the same kind of algebra as that of the de Rham cohomology operators of differential geometry defined on a compact manifold [1, 5, 6, 11]. The existence of some subtle discrete symmetries has been shown to correspond to the Hodge $*$ duality operation of the ordinary differential geometry for the case of 2D one-form and 4D two-form Abelian gauge theories [6, 11]. Under these discrete symmetries, the (anti-)BRST- and (anti-)co-BRST symmetry transformations are related to one another in exactly the same way as the cohomological operators d and $\delta = \pm * \mathrm{d} *$ are related to each other [6]. In our present paper, we show that a proper generalization of these discrete symmetries for the case of a 2D free $U(1)$ gauge theory in the superfield formulation (together with the idea of Hodge decomposition for the 2D fermionic vector fields) provides a logical origin for the existence of the 'off-shell' nilpotent, local, covariant and continuous (anti-)BRST- and (anti-)co-BRST symmetries. The 'on-shell' version of these results has already been obtained in [12]. We lay emphasis on the fact that, in our present attempt, we concentrate *only* on the 'off-shell' version of these nilpotent symmetries which are more general than their 'on-shell' counterparts. It is evident that we do not exploit the classical equations of motion *anywhere* in our present discussion for the off-shell nilpotent (anti-)BRST and (anti-)co-BRST symmetries.

One of the most popular geometrical approaches to the BRST formalism (in the language of the Maurer-Cartan equation and the translation generators) is the superfield formulation [13–18]. In this superfield formulation, so far, only the mathematical power of the super exterior derivative $\tilde{\mathrm{d}}$, together with the idea of the horizontality condition[2] on the two-form constructed with it, has been tapped which leads to the interpretation of the nilpotent (anti-)BRST charges as the generators for translations along the Grassmannian directions of the supermanifold for the one-form gauge theory [13–15]. In view of the generality and applicability of this formulation to any arbitrary n-form ($n = 1, 2 \ldots$) gauge theories in any arbitrary spacetime dimension [15–18], it is an interesting endeavour to provide the geometrical interpretation for the nilpotent (anti-)dual BRST symmetries and a bosonic symmetry (generated by the Casimir operator) [1–6, 11] in the language of other super de Rham cohomology operators $\tilde{\delta} = \pm \star \tilde{\mathrm{d}} \star$ and $\tilde{\Delta} = \tilde{\mathrm{d}}\tilde{\delta} + \tilde{\delta}\tilde{\mathrm{d}}$ defined on the four (2+2)-dimensional

---

[1] This equation is the analogue of the HDT which states that any arbitrary n-form f_n ($n = 0, 1, 2, \ldots$), on a compact manifold, can be decomposed into a harmonic form ω_n ($\Delta\omega_n = 0$, $\mathrm{d}\omega_n = 0$, $\delta\omega_n = 0$), an exact form $\mathrm{d}g_{n-1}$ and a co-exact form δh_{n+1} as: $f_n = \omega_n + \mathrm{d}g_{n-1} + \delta h_{n+1}$ where $\delta(= \pm * \mathrm{d}*)$ is the Hodge dual of d (with $\mathrm{d}^2 = 0$, $\delta^2 = 0$; $* =$ Hodge duality operation) and Laplacian Δ is defined as $\Delta = (\mathrm{d} + \delta)^2 = \mathrm{d}\delta + \delta\mathrm{d}$ [7–10]. The set (d, δ, Δ) is known as the de Rham cohomology operators of differential geometry and they obey the algebra: $\mathrm{d}^2 = \delta^2 = 0$, $[\Delta, \mathrm{d}] = [\Delta, \delta] = 0$, $\Delta = \{\mathrm{d}, \delta\} = (\mathrm{d} + \delta)^2$.

[2] This condition is referred to as the 'soul flatness' in [18] implying the flatness of the curvature (two-form) tensor in the Grassmannian directions of the supermanifold.

compact supermanifold. Here $\tilde{\delta}$ and $\tilde{\Delta}$ are the super co-exterior derivative and super Laplacian operator, respectively. To the best of our knowledge, these super cohomological operators have not yet been exploited in the context of (geometrical) superfield approach to BRST formalism, BRST cohomology and HDT in the quantum Hilbert space of states for any gauge theory in any spacetime dimension. *One of the central themes of the present paper is to make an attempt towards this goal and to show that useful symmetries do emerge for the 2D free Abelian gauge theory when we exploit these geometrical (cohomological) super operators together with the idea of the generalized version of horizontality condition.* We demonstrate that the off-shell nilpotent (anti-)dual BRST symmetries (generated by (anti-)co-BRST charges) emerge when we exploit the super co-exterior derivative $\tilde{\delta} = \pm\star\tilde{d}\star$ together with an analogue of the 'horizontality' condition. Geometrically, the nilpotent and conserved (anti-)co-BRST charges (similar to their counterpart (anti-)BRST charges [15]) turn out to be the translation generators along the Grassmannian directions[3] of the supermanifold. When we exploit the mathematical power of super Laplacian operator $\tilde{\Delta} = \tilde{d}\tilde{\delta} + \tilde{\delta}\tilde{d}$ in this context, we obtain a bosonic symmetry transformation for the $U(1)$ gauge field *alone* and ghost fields do not transform at all. As expected, the Noether conserved charge corresponding to this symmetry emerges as the generator for the above symmetry transformations. In the language of the geometry on the supermanifold, this bosonic charge (Casimir operator) turns out to be the generator of translations along the $\theta\bar{\theta}$-direction of the supermanifold. In fact, this bosonic direction is equivalent to a couple of successive translations along the Grassmannian (θ and $\bar{\theta}$) directions of the supermanifold.

This paper is organized as follows. In section 2, we set up the notation and recapitulate the bare essentials of our earlier works [1–6] for the 2D free Abelian gauge theory in the framework of Lagrangian formulation. This is followed by the derivation of the (anti-)BRST symmetries in the framework of superfield formalism in section 3 by exploiting the idea of 'horizontality' condition [15]. In section 4, we derive the (anti-)dual BRST symmetries by exploiting the super dual exterior derivative $\tilde{\delta}$ together with a restriction ($\tilde{\delta}\tilde{A} = \delta A$) which is the analogue of the horizontality condition. Section 5 is devoted to the discussion of some new discrete symmetries that are present in the superfield formulation. In section 6, we derive a bosonic symmetry by exploiting the mathematical power of $\tilde{\Delta}$ together with a 'horizontality type' restriction wrt this operator. Finally, in section 7, we make some concluding remarks and point out some future directions that can be pursued later.

2. Preliminary: (anti-)BRST and (anti-)co-BRST symmetries

We begin with the BRST invariant Lagrangian density ($\mathcal{L}_b$) for the flat Minkowskian two (1+1)-dimensional[4] free Abelian gauge theory in the Feynman gauge (see, e.g., [18–20])

$$\mathcal{L}_b = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} - \tfrac{1}{2}(\partial \cdot A)^2 - \mathrm{i}\partial_\mu\bar{C}\partial^\mu C \equiv \tfrac{1}{2}E^2 - \tfrac{1}{2}(\partial \cdot A)^2 - \mathrm{i}\partial_\mu\bar{C}\partial^\mu C \qquad (2.1)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ (with $F_{01} = E$ as electric field) is the field strength tensor derived from the one-form $A = \mathrm{d}x^\mu A_\mu$ (with A_μ as the vector potential) by application of the exterior derivative d (i.e. $F = \mathrm{d}A$). On the other hand, the gauge-fixing term $(\partial \cdot A)$ is derived from the one-form $A = \mathrm{d}x^\mu A_\mu$ by application of the co-exterior derivative δ (i.e. $(\partial \cdot A) = \delta A$, $\delta = -*\mathrm{d}*$). Thus, two-form $F = \mathrm{d}A$ and zero-form $(\partial \cdot A) = \delta A$ are 'Hodge'

---

[3] Even though (anti-)BRST and (anti-)co-BRST charges generate translations along the Grassmannian directions, there is a clear distinction between them. To be precise, these charges generate translations for the fermionic (anti-)ghost fields along *different* Grassmannian directions of the supermanifold (cf section 4).

[4] We adopt here the notations and conventions in which the 2D flat Minkowski metric is : $\eta_{\mu\nu} = \mathrm{diag}\,(+1, -1)$ and $\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu = \partial_0\partial_0 - \partial_1\partial_1$, $F_{01} = -\varepsilon^{\mu\nu}\partial_\mu A_\nu = \partial_0 A_1 - \partial_1 A_0 = E = F^{10}$, $\varepsilon_{01} = \varepsilon^{10} = +1$.

dual to each other. The (anti-)ghost fields $(\bar{C})C$ are anti-commuting $(\bar{C}^2 = C^2 = 0, C\bar{C} = -\bar{C}C)$ in nature. One can linearize the kinetic energy term and the gauge-fixing term by introducing auxiliary fields $\mathcal{B}$ and B, as

$$\mathcal{L}_B = \mathcal{B}E - \tfrac{1}{2}\mathcal{B}^2 + B(\partial \cdot A) + \tfrac{1}{2}B^2 - \mathrm{i}\partial_\mu \bar{C}\partial^\mu C. \tag{2.2}$$

This Lagrangian density respects the following off-shell nilpotent $\left(s_b^2 = 0, s_d^2 = 0\right)$ BRST (s_b) and dual-BRST (s_d) symmetry transformations[5]

$$s_b A_\mu = \partial_\mu C \qquad s_b C = 0 \qquad s_b \mathcal{B} = 0 \qquad s_b E = 0$$
$$s_b \bar{C} = \mathrm{i}B \qquad s_b B = 0 \qquad s_b(\partial \cdot A) = \square C \tag{2.3}$$

$$s_d A_\mu = -\varepsilon_{\mu\nu}\partial^\nu \bar{C} \qquad s_d C = -\mathrm{i}\mathcal{B} \qquad s_d \mathcal{B} = 0$$
$$s_d E = \square \bar{C} \qquad s_d \bar{C} = 0 \qquad s_d B = 0 \qquad s_d(\partial \cdot A) = 0. \tag{2.4}$$

The corresponding off-shell nilpotent $\left(s_{ab}^2 = 0, s_{ad}^2 = 0\right)$ anti-BRST (s_{ab}) symmetries (with $s_b s_{ab} + s_{ab} s_b = 0$) and anti-co-BRST (s_{ad}) symmetries (with $s_d s_{ad} + s_{ad} s_d = 0$) are

$$s_{ab} A_\mu = \partial_\mu \bar{C} \qquad s_{ab}\bar{C} = 0 \qquad s_{ab}\mathcal{B} = 0 \qquad s_{ab}E = 0$$
$$s_{ab} C = -\mathrm{i}B \qquad s_{ab}B = 0 \qquad s_{ab}(\partial \cdot A) = \square \bar{C} \tag{2.5}$$

$$s_{ad} A_\mu = -\varepsilon_{\mu\nu}\partial^\nu C \qquad s_{ad}\bar{C} = \mathrm{i}\mathcal{B} \qquad s_{ad}\mathcal{B} = 0$$
$$s_{ad} E = \square C \qquad s_{ad} C = 0 \qquad s_{ad} B = 0 \qquad s_{ad}(\partial \cdot A) = 0. \tag{2.6}$$

The anti-commutators of the above symmetries lead to the definition of a bosonic symmetry $s_w = \{s_b, s_d\} = \{s_{ab}, s_{ad}\}, s_w^2 \neq 0$. Under this symmetry, the (anti-)ghost fields do not transform and only the $U(1)$ gauge field transforms as [1, 2, 6]

$$s_w A_\mu = (\partial_\mu \mathcal{B} + \varepsilon_{\mu\nu}\partial^\nu B) \equiv \varepsilon_{\mu\nu}(\partial^\nu B + \varepsilon^{\nu\lambda}\partial_\lambda \mathcal{B}) \qquad s_w B = 0$$
$$s_w C = 0 \qquad s_w \bar{C} = 0 \qquad s_w \mathcal{B} = 0 \qquad s_w(\partial \cdot A) = \square \mathcal{B} \qquad s_w E = -\square B. \tag{2.7}$$

It can be seen that the transformation for the $U(1)$ gauge field A_μ is just equal to its own equation of motion: $\partial_\mu \mathcal{B} + \varepsilon_{\mu\nu}\partial^\nu B(=0)$. The substitutions $\mathcal{B} = E, B = -(\partial \cdot A)$ make the above equation of motion equal to $-\varepsilon_{\mu\nu}\square A^\nu(=0)$. In terms of the vector fields A_μ alone, now we have the transformation $\tilde{s}_w A_\mu = -\varepsilon_{\mu\nu}\square A^\nu$. Thus, we note that all the transformations s_w in (2.7) (and their analogues $\tilde{s}_w$) are trivially zero on the on-shell[6]. All the above continuous symmetry transformations can be concisely expressed, in terms of generators Q_r (see, e.g., section 7 for explicit local expressions) and generic field ϕ, as

$$s_r \phi = -\mathrm{i}[\phi, Q_r]_\pm \qquad r = b, ab, d, ad, w \tag{2.8}$$

where Q_r are the conserved Noether charges derived from the conserved currents and brackets $[\,,\,]_\pm$ stand for the (anti-)commutators for ϕ being (fermionic)bosonic in nature.

　　　Now we shall dwell a bit on the presence of some interesting discrete symmetries in the theory. It can be seen from (2.3) and (2.5) that $s_b \leftrightarrow s_{ab}$ for the discrete transformations: $C \leftrightarrow \bar{C}, B \leftrightarrow -B$. In a similar fashion, it is clear from (2.4) and (2.6) that $s_d \leftrightarrow s_{ad}$ for the presence of the discrete transformations: $C \leftrightarrow \bar{C}, \mathcal{B} \leftrightarrow -\mathcal{B}$. In other words, the discrete symmetries: $C \leftrightarrow \bar{C}, B \leftrightarrow -B, \mathcal{B} \leftrightarrow -\mathcal{B}, A_\mu \leftrightarrow A_\mu$, connect the BRST to anti-BRST and

---

[5] We follow here the notation adopted in [21]. In fact, a BRST transformation δ_B is the product of an anti-commuting constant parameter ζ and the transformation s_b (i.e. $\delta_B = \zeta s_b$) in its full glory.

[6] We shall be exploiting this observation in sections 6 and 7. It will be noted, however, that $\tilde{s}_w$ (with $\tilde{s}_w C = 0, \tilde{s}_w \bar{C} = 0, \tilde{s}_w A_\mu = -\varepsilon_{\mu\nu}\square A^\nu$) is a symmetry transformation for the Lagrangian density (2.1) and is the analogue of the symmetry s_w (cf (2.7)) for the Lagrangian density (2.2).

the co-BRST to anti-co-BRST symmetries and vice versa. Yet another interesting discrete symmetry transformations [1, 2, 6]

$$
\begin{array}{llll}
C \to \pm i\bar{C} & E \to \pm i(\partial \cdot A) & A_\mu \to A_\mu & \partial_\mu \to \pm i\varepsilon_{\mu\nu}\partial^\nu \\
\bar{C} \to \pm iC & (\partial \cdot A) \to \pm iE & \mathcal{B} \to \mp i B & B \to \mp i\mathcal{B}
\end{array}
\tag{2.9}
$$

connect s_b with s_d (as well as s_{ab} with s_{ad}) and *leave the Lagrangian density* (2.2) *form-invariant* [6]. The topological nature of this theory has been shown by demonstrating that the Lagrangian density in (2.1) can be written, modulo some total derivatives, as the sum of BRST and dual BRST anti-commutators [1, 2, 6]

$$
\begin{aligned}
\mathcal{L}_b &= \{Q_d, T_1\} + \{Q_b, T_2\} \equiv s_d(iT_1) + s_b(iT_2) \\
\mathcal{L}_b &= \{Q_{ad}, P_1\} + \{Q_{ab}, P_2\} \equiv s_{ad}(iP_1) + s_{ab}(iP_2)
\end{aligned}
\tag{2.10}
$$

where the local expressions for $T_{(1,2)}$ and $P_{(1,2)}$ are: $T_1 = \frac{1}{2}EC$, $T_2 = -\frac{1}{2}(\partial \cdot A)\bar{C}$, $P_1 = -\frac{1}{2}E\bar{C}$, $P_2 = \frac{1}{2}(\partial \cdot A)C$. Similarly, the off-shell nilpotent (anti-)BRST and (anti-)dual BRST invariant Lagrangian density (2.2) can be written in two different ways as: (i) the sum of the kinetic energy term and an (anti-)BRST invariant part or (ii) the sum of the gauge-fixing term and an (anti-)dual BRST invariant part. These alternative ways of expressing the Lagrangian density are

$$
\mathcal{L}_B = \mathcal{B}E - \tfrac{1}{2}\mathcal{B}^2 + s_b s_{ab}\left(\tfrac{i}{2}A^2 - \tfrac{1}{2}\bar{C}C\right) \equiv B(\partial \cdot A) + \tfrac{1}{2}B^2 + s_d s_{ad}\left(\tfrac{i}{2}A^2 - \tfrac{1}{2}\bar{C}C\right).
\tag{2.11}
$$

The topological invariants for this theory and their well-known recursion relations have been computed on a 2D compact manifold [1, 2, 6]. The energy–momentum tensor has been shown to be the sum of BRST and co-BRST anti-commutators [2, 6].

3. Superfield formulation for (anti-)BRST symmetry transformations

We start off with a four (2+2)-dimensional supermanifold that is parametrized by two c-number commuting (bosonic) spacetime coordinates x^μ ($\mu = 0, 1$) and two Grassmann variables θ and $\bar{\theta}$. We define a supervector superfield v_s on this supermanifold as [15][7]

$$
v_s = (\Phi_\mu(x, \theta, \bar{\theta}), \eta(x, \theta, \bar{\theta}), \bar{\eta}(x, \theta, \bar{\theta}))
\tag{3.1}
$$

where $\Phi_\mu(x, \theta, \bar{\theta})$ are the bosonic (even) superfields and $\eta(x, \theta, \bar{\theta})$, $\bar{\eta}(x, \theta, \bar{\theta})$ are the fermionic (odd) superfields which constitute the supermultiplet of the supervector superfield v_s. In terms of the superspace coordinates $x^\mu, \theta, \bar{\theta}$, the component superfields can be expanded as

$$
\begin{aligned}
\Phi_\mu(x, \theta, \bar{\theta}) &= A_\mu(x) + \theta \bar{R}_\mu(x) + \bar{\theta} R_\mu(x) + i\theta\bar{\theta} S_\mu(x) \\
\eta(x, \theta, \bar{\theta}) &= C(x) + i\theta \bar{B}(x) + i\bar{\theta}\mathcal{B}(x) + i\theta\bar{\theta} s(x) \\
\bar{\eta}(x, \theta, \bar{\theta}) &= \bar{C}(x) + i\theta \bar{\mathcal{B}}(x) + i\bar{\theta} B(x) + i\theta\bar{\theta} \bar{s}(x)
\end{aligned}
\tag{3.2}
$$

where field variables $A_\mu(x) \equiv A_\mu(x, 0, 0)$; $C(x) \equiv C(x, 0, 0)$, etc are functions of only spacetime variables. It will be noted that the local fields: $A_\mu(x), S_\mu(x), B(x), \bar{B}(x), \mathcal{B}(x), \bar{\mathcal{B}}(x)$ are bosonic (even) in nature whereas the local fermionic (odd) fields in the theory are: $R_\mu(x), \bar{R}_\mu(x), C(x), \bar{C}(x), s(x), \bar{s}(x)$. The super exterior derivative $\tilde{d} = dz^M \partial_M$ on this supermanifold is defined in terms of the superspace differentials $(dx^\mu, d\theta, d\bar{\theta})$ as [22]:

$$
\tilde{d} = dx^\mu \partial_\mu + d\theta \partial_\theta + d\bar{\theta} \partial_{\bar{\theta}}
\tag{3.3}
$$

---

[7] We follow the field notations of [13, 15, 17] but be consistent with the conventions adopted in [22] for the definition of the super derivatives and super differential forms on a compact supermanifold.

where

$$\tilde{d} = dz^M \frac{\partial}{\partial z^M} \qquad \tilde{d}^2 = 0 \qquad z^M = (x^\mu, \theta, \bar{\theta})$$
$$\partial_\mu = \frac{\partial}{\partial x^\mu} \qquad \partial_\theta = \frac{\partial}{\partial \theta} \qquad \partial_{\bar{\theta}} = \frac{\partial}{\partial \bar{\theta}}. \tag{3.4}$$

A connection super one-form on the supermanifold can be defined in terms of the component superfields of the supervector superfield v_s as

$$\tilde{A} = dx^\mu \, \Phi_\mu(x, \theta, \bar{\theta}) + d\theta \, \bar{\eta}(x, \theta, \bar{\theta}) + d\bar{\theta} \, \eta(x, \theta, \bar{\theta}). \tag{3.5}$$

The Maurer–Cartan equation that defines the curvature two-form $\tilde{F}$ from the connection one-form $\tilde{A}$ and the exterior derivative $\tilde{d}$ on the supermanifold, in its most general form, is

$$\tilde{F} = \tilde{d}\tilde{A} + \tilde{A} \wedge \tilde{A} \equiv \tilde{d}\tilde{A} + \tfrac{1}{2}[\tilde{A}, \tilde{A}]. \tag{3.6}$$

For Abelian $U(1)$ gauge theory, however, the last term in the above equation is zero (i.e., $\tilde{A} \wedge \tilde{A} = \frac{1}{2}[\tilde{A}, \tilde{A}] = 0$). The horizontality condition imposes the restriction that the components of the curvature two-form $\tilde{F} = \tilde{d}\tilde{A}$ for the Abelian gauge theory (with $\{d\theta(d\bar{\theta}), \theta(\bar{\theta}\} = 0$)

$$\tilde{F} = \tilde{d}\tilde{A} = (dx^\mu \wedge dx^\nu)(\partial_\mu \Phi_\nu) + (dx^\mu \wedge d\theta)(\partial_\mu \bar{\eta} - \partial_\theta \Phi_\mu) - (d\theta \wedge d\theta)(\partial_\theta \bar{\eta})$$
$$+ (dx^\mu \wedge d\bar{\theta})(\partial_\mu \eta - \partial_{\bar{\theta}} \Phi_\mu) - (d\theta \wedge d\bar{\theta})(\partial_\theta \bar{\eta} + \partial_{\bar{\theta}} \eta) - (d\bar{\theta} \wedge d\bar{\theta})(\partial_{\bar{\theta}} \eta)$$

must vanish along the Grassmannian directions $(\theta, \bar{\theta})$ of the supermanifold. This ultimately amounts to the following condition (which is nothing but $\tilde{d}\tilde{A} = dA$)

$$\tilde{F} \equiv \tfrac{1}{2}(dz^M \wedge dz^N)\tilde{F}_{MN} = \tfrac{1}{2}(dx^\mu \wedge dx^\nu)F_{\mu\nu} \equiv F \tag{3.7}$$

where the wedge product $dz^M \wedge dz^N$, in the component form of the superspace variables, is: $dx^\mu \wedge dx^\nu = -dx^\nu \wedge dx^\mu$, $d\theta \wedge d\bar{\theta} = d\bar{\theta} \wedge d\theta$, $dx^\mu \wedge d\theta = -d\theta \wedge dx^\mu$, etc. In the following, the above horizontality condition (3.7) leads to

$$\begin{aligned}
\partial_\theta \bar{\eta} = 0 \qquad &\rightarrow \qquad \bar{\mathcal{B}} = 0 \qquad && \bar{s} = 0 \\
\partial_{\bar{\theta}} \eta = 0 \qquad &\rightarrow \qquad \mathcal{B} = 0 \qquad && s = 0 \\
\partial_\theta \eta = -\partial_{\bar{\theta}} \bar{\eta} \qquad &\rightarrow \qquad B + \bar{B} = 0 \\
\partial_\mu \eta = \partial_{\bar{\theta}} \Phi_\mu \qquad &\rightarrow \qquad R_\mu = \partial_\mu C \qquad && S_\mu = -\partial_\mu \bar{B} \equiv \partial_\mu B \\
\partial_\mu \bar{\eta} = \partial_\theta \Phi_\mu \qquad &\rightarrow \qquad \bar{R}_\mu = \partial_\mu \bar{C} \qquad && S_\mu = \partial_\mu B
\end{aligned} \tag{3.8}$$

which satisfy the other conditions, namely

$$\partial_\mu \bar{R}_\nu - \partial_\nu \bar{R}_\mu = 0 \qquad \partial_\mu R_\nu - \partial_\nu R_\mu = 0 \qquad \partial_\mu S_\nu - \partial_\nu S_\mu = 0. \tag{3.9}$$

It is clear (cf section 2) that the (anti-)BRST charges are the generators (cf (2.8)) of translations along the Grassmannian directions (i.e., $\text{Lim}_{\theta, \bar{\theta} \to 0} \frac{\partial}{\partial \bar{\theta}(\theta)} \Sigma(x, \theta, \bar{\theta}) = i[Q_{(a)b}, \Lambda]_\pm$, for $\Sigma = \eta, \bar{\eta}, \Phi_\mu$ and corresponding $\Lambda = C, \bar{C}, A_\mu$) of the supermanifold as the expansions in (3.2) can be recast in terms of $s_{(a)b}$ as

$$\begin{aligned}
\Phi_\mu(x, \theta, \bar{\theta}) &= A_\mu(x) + \theta(s_{ab} A_\mu(x)) + \bar{\theta}(s_b A_\mu(x)) + \theta\bar{\theta}(s_b s_{ab} A_\mu(x)) \\
\eta(x, \theta, \bar{\theta}) &= C(x) + \theta(s_{ab} C(x)) + \bar{\theta}(s_b C(x)) + \theta\bar{\theta}(s_b s_{ab} C(x)) \\
\bar{\eta}(x, \theta, \bar{\theta}) &= \bar{C}(x) + \theta(s_{ab} \bar{C}(x)) + \bar{\theta}(s_b \bar{C}(x)) + \theta\bar{\theta}(s_b s_{ab} \bar{C}(x))
\end{aligned} \tag{3.10}$$

which finally amounts to the following expansion:

$$\begin{aligned}
\Phi_\mu(x, \theta, \bar{\theta}) &= A_\mu(x) + \theta\partial_\mu \bar{C}(x) + \bar{\theta}\partial_\mu C(x) + i\theta\bar{\theta}\partial_\mu B(x) \\
\eta(x, \theta, \bar{\theta}) &= C(x) + i\theta \bar{B}(x) \equiv C(x) - i\theta B(x) \\
\bar{\eta}(x, \theta, \bar{\theta}) &= \bar{C}(x) + i\bar{\theta} B(x).
\end{aligned} \tag{3.11}$$

This, in a nutshell, provides the origin for the existence of (anti-)BRST symmetries in the framework of superfield formulation.

4. Superfield approach to (anti-)dual BRST transformations

On an even dimensional *ordinary* spacetime manifold, it is obvious that the operation of $\delta = -*\mathrm{d}*$[8] on the one-form $A = \mathrm{d}x^\mu A_\mu$ leads to the zero-form as none other than the gauge-fixing term $(\partial \cdot A)$ [7, 8]:

$$\delta A = -*\mathrm{d}*A = (\partial \cdot A) \tag{4.1}$$

where the Hodge $*$ duality operation on the ordinary differentials in the flat Minkowskian two-dimensional spacetime is: $*(\mathrm{d}x^\mu) = \varepsilon^{\mu\nu}(\mathrm{d}x_\nu)$, $*(\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu) = \varepsilon^{\mu\nu}$. Now, on the connection super one-form $\tilde{A}$ defined on a (2+2)-dimensional supermanifold, we apply the super co-exterior derivative $\tilde{\delta}$ and exploit the following condition:

$$\tilde{\delta}\tilde{A} = -\star\tilde{\mathrm{d}}\star\tilde{A} = (\partial \cdot A). \tag{4.2}$$

In words, this equation amounts to restricting the zero-form superscalar superfield $(\tilde{\delta}\tilde{A})$, that emerges after the application of $\tilde{\delta}$ on the connection super one-form $\tilde{A}$, to the zero-form gauge-fixing term $(\delta A = (\partial \cdot A))$ defined on the ordinary flat 2D Minkowskian spacetime manifold. This restriction is the analogue of horizontality condition wrt $\tilde{\delta}$. Here the Hodge $\star$ duality operation is defined on the (2+2)-dimensional compact supermanifold. The basic superspace differentials transform under this operation as

$$\begin{array}{lll}
\star(\mathrm{d}x^\mu) = \varepsilon^{\mu\nu}(\mathrm{d}x_\nu) & \star(\mathrm{d}\theta) = (\mathrm{d}\bar{\theta}) & \star(\mathrm{d}\bar{\theta}) = (\mathrm{d}\theta) \\
\star(\mathrm{d}\theta \wedge \mathrm{d}\theta) = s^{\theta\theta} & \star(\mathrm{d}\bar{\theta} \wedge \mathrm{d}\bar{\theta}) = s^{\bar{\theta}\bar{\theta}} & \star(\mathrm{d}\theta \wedge \mathrm{d}\bar{\theta}) = s^{\theta\bar{\theta}} \quad (4.3) \\
\star(\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu) = \varepsilon^{\mu\nu} & \star(\mathrm{d}x^\mu \wedge \mathrm{d}\theta) = \varepsilon^{\mu\theta} & \star(\mathrm{d}x^\mu \wedge \mathrm{d}\bar{\theta}) = \varepsilon^{\mu\bar{\theta}}
\end{array}$$

where $\varepsilon^{\mu\theta(\bar{\theta})} = -\varepsilon^{\theta(\bar{\theta})\mu}$ are anti-symmetric and $s^{\theta\theta} = s^{\bar{\theta}\theta}$ etc are symmetric. The expansion of the lhs of (4.2) defines a superscalar (zero-form) superfield

$$\begin{aligned}
\tilde{\delta}\tilde{A} = {} & (\partial \cdot \Phi) + s^{\theta\theta}(\partial_\theta \eta) + s^{\bar{\theta}\bar{\theta}}(\partial_{\bar{\theta}}\bar{\eta}) + s^{\theta\bar{\theta}}(\partial_\theta \bar{\eta} + \partial_{\bar{\theta}}\eta) \\
& - \varepsilon^{\mu\theta}\left(\partial_\mu \eta + \varepsilon_{\mu\nu}\partial_\theta \Phi^\nu\right) - \varepsilon^{\mu\bar{\theta}}\left(\partial_\mu \bar{\eta} + \varepsilon_{\mu\nu}\partial_{\bar{\theta}}\Phi^\nu\right)
\end{aligned}$$

and its subsequent equality with the ordinary gauge-fixing term in the rhs (due to the analogue of the horizontality requirement) leads to

$$\begin{array}{llll}
\partial_{\bar{\theta}}\bar{\eta} = 0 & \rightarrow & B = 0 & \bar{s} = 0 \\
\partial_\theta \eta = 0 & \rightarrow & \bar{B} = 0 & s = 0 \\
\partial_\theta \bar{\eta} = -\partial_{\bar{\theta}}\eta & \rightarrow & \mathcal{B} + \bar{\mathcal{B}} = 0 & (4.4) \\
\partial_\mu \bar{\eta} = -\varepsilon_{\mu\nu}\partial_{\bar{\theta}}\Phi^\nu & \rightarrow & R_\mu = -\varepsilon_{\mu\nu}\partial^\nu \bar{C} & S_\mu = +\varepsilon_{\mu\nu}\partial^\nu \bar{\mathcal{B}} \equiv -\varepsilon_{\mu\nu}\partial^\nu \mathcal{B} \\
\partial_\mu \eta = -\varepsilon_{\mu\nu}\partial_\theta \Phi^\nu & \rightarrow & \bar{R}_\mu = -\varepsilon_{\mu\nu}\partial^\nu C & S_\mu = -\varepsilon_{\mu\nu}\partial^\nu \mathcal{B}.
\end{array}$$

The above results also satisfy the following conditions:

$$\partial \cdot \bar{R} = 0 \qquad \partial \cdot R = 0 \qquad \partial \cdot S = 0. \tag{4.5}$$

It will be noted that (4.4) *can be directly obtained from* (3.8) *by exploiting the duality symmetry* (2.9). In the language of the expansion of the superfields, we have obtained

$$\begin{aligned}
\Phi_\mu(x, \theta, \bar{\theta}) &= A_\mu(x) - \theta \varepsilon_{\mu\nu}\partial^\nu C(x) - \bar{\theta}\varepsilon_{\mu\nu}\partial^\nu \bar{C}(x) - \mathrm{i}\theta\bar{\theta}\varepsilon_{\mu\nu}\partial^\nu \mathcal{B}(x) \\
\eta(x, \theta, \bar{\theta}) &= C(x) + \mathrm{i}\bar{\theta}\mathcal{B}(x) \\
\bar{\eta}(x, \theta, \bar{\theta}) &= \bar{C}(x) + \mathrm{i}\theta\bar{\mathcal{B}}(x) \equiv \bar{C}(x) - \mathrm{i}\theta\mathcal{B}(x).
\end{aligned} \tag{4.6}$$

---

[8] In any arbitrary flat Minowskian even *D*-dimensional ordinary spacetime manifolds, the ordinary dual-exterior derivative δ is: $\delta = -*\mathrm{d}*$. In general, an inner-product of a *n*-form in *D*-dimensional spacetime manifold leads to $\delta = (-1)^{Dn+D+1}*\mathrm{d}*$. Thus, for odd *D*, we have $\delta = (-1)^n*\mathrm{d}*$ (see, e.g. [7]).

This result can be further written in terms of the (anti-)dual BRST $s_{(a)d}$ symmetries (cf equation (2.4) and (2.6)) obtained in section 2, as follows:

$$\Phi_\mu(x, \theta, \bar{\theta}) = A_\mu(x) + \theta(s_{ad} A_\mu(x)) + \bar{\theta}(s_d A_\mu(x)) - \theta\bar{\theta}(s_d s_{ad} A_\mu(x))$$
$$\eta(x, \theta, \bar{\theta}) = C(x) + \theta(s_{ad} C(x)) - \bar{\theta}(s_d C(x)) + \theta\bar{\theta}(s_d s_{ad} C(x)) \qquad (4.7)$$
$$\bar{\eta}(x, \theta, \bar{\theta}) = \bar{C}(x) - \theta(s_{ad}\bar{C}(x)) + \bar{\theta}(s_d\bar{C}(x)) + \theta\bar{\theta}(s_d s_{ad}\bar{C}(x)).$$

These equations (4.5)–(4.7), finally establish the origin of the existence of (anti-)dual BRST symmetries (discussed in section 2) in the framework of superfield formalism. It is evident that application of a single restriction (i.e., the analogue of the horizontality condition) with the super co-exterior derivative (operating on $\tilde{A}$) leads to the derivation of two nilpotent (anti-) dual BRST symmetries for the 2D free Abelian gauge theory. It is worthwhile to compare and contrast transformations (3.11) *vis-a-vis* transformations (4.6) that are generated by (anti-)BRST and (anti-)co-BRST charges $Q_{(a)b}$ and $Q_{(a)d}$, respectively. These are: (i) (anti-)BRST and (anti-)co-BRST transformations correspond to translations along the θ and $\bar{\theta}$ directions of the supermanifold for bosonic and fermionic fields of the theory. (ii) For the bosonic field A_μ, the above nilpotent charges generate translations along the θ, $\bar{\theta}$ and $\theta\bar{\theta}$ directions of the supermanifold. (iii) For the fermionic fields C and $\bar{C}$, the translations are either along the θ or $\bar{\theta}$ directions of the supermanifold. (iv) For the (anti-)ghost fields $(\bar{C})\,C$, the (anti-)BRST charges generate translations along $(\bar{\theta})\,\theta$ directions of the supermanifold. In contrast, for the same fields, the (anti-)co-BRST charges generate translations along $(\theta)\,\bar{\theta}$ directions. (v) It is the mathematical power of $\tilde{d}$ (together with the Maurer–Cartan equation) that produces (anti-)BRST transformations. On the other hand, the derivation of the (anti-)co-BRST transformations depends entirely on the mathematical power of $\tilde{\delta}$. (vi) The mapping between (anti-)BRST and (anti-)co-BRST charges on the one hand and the super cohomological operators $\tilde{d}$ and $\tilde{\delta}$ on the other hand, is: $Q_{(a)b} \Leftrightarrow \tilde{d}$ and $Q_{(a)d} \Leftrightarrow \tilde{\delta}$. (vii) The local conserved charges $Q_{(a)b}$ and $Q_{(a)d}$ are connected with the *ordinary* cohomological operators d and δ through the mappings: $(Q_b, Q_{ad}) \Leftrightarrow$ d, $(Q_d, Q_{ab}) \Leftrightarrow \delta$ [1, 2, 6].

5. Discrete symmetry transformations in superfield formulation

We have discussed the existence of a few discrete symmetries in section 2. They are interesting in the sense that they enable us to gain some insights into various connections which exist among all BRST type symmetries that are present in the theory. In fact, it can be checked that the discrete transformations: $C \leftrightarrow \bar{C}, B \leftrightarrow -B, -\mathcal{B} \leftrightarrow -\mathcal{B}$ (that connect BRST- to anti-BRST and co-BRST- to anti-co-BRST symmetries) can be generalized *vis-a-vis* super expansion (3.2) in terms of superspace coordinates and component fields as

$$C \leftrightarrow \bar{C} \qquad B \leftrightarrow -B \qquad \mathcal{B} \leftrightarrow -\mathcal{B} \qquad \theta \leftrightarrow \bar{\theta} \qquad R_\mu \leftrightarrow \bar{R}_\mu \qquad S_\mu \leftrightarrow -S_\mu \qquad (5.1)$$

under which the superfields transform as: $\Phi_\mu(x, \theta, \bar{\theta}) \leftrightarrow \Phi_\mu(x, \theta, \bar{\theta}), \eta(x, \theta, \bar{\theta}) \leftrightarrow \bar{\eta}(x, \theta, \bar{\theta})$. It will be noted that for this discussion, we have taken $s(x) = \bar{s}(x) = 0$ and $B + \bar{B} = 0, \mathcal{B} + \bar{\mathcal{B}} = 0$ in expansion (3.2), i.e.,

$$\Phi_\mu(x, \theta, \bar{\theta}) = A_\mu(x) + \theta\bar{R}_\mu(x) + \bar{\theta}R_\mu(x) + i\theta\bar{\theta}S_\mu(x)$$
$$\eta(x, \theta, \bar{\theta}) = C(x) - i\theta B(x) + i\bar{\theta}\mathcal{B}(x)$$
$$\bar{\eta}(x, \theta, \bar{\theta}) = \bar{C}(x) - i\theta\mathcal{B}(x) + i\bar{\theta}B(x)$$

due to the fact that these restrictions emerge when we show the existence of (anti-)BRST and (anti-)co-BRST symmetries in the framework of superfield formulation. The Hodge decomposed versions for the 2D vector fields $(R_\mu, \bar{R}_\mu, S_\mu)$, consistent with the transformations (5.1), can be written in terms of the ghost and auxiliary fields as

$$R_\mu = \partial_\mu C - \varepsilon_{\mu\nu}\partial^\nu\bar{C} \qquad \bar{R}_\mu = \partial_\mu\bar{C} - \varepsilon_{\mu\nu}\partial^\nu C \qquad S_\mu = \partial_\mu B - \varepsilon_{\mu\nu}\partial^\nu\mathcal{B} \qquad (5.2)$$

which obey $R_\mu \leftrightarrow \bar{R}_\mu, S_\mu \leftrightarrow -S_\mu$ under $C \leftrightarrow \bar{C}, B \leftrightarrow -B, \mathcal{B} \leftrightarrow -\mathcal{B}$. Now we wish to concentrate on the discrete symmetry transformations (2.9). This symmetry can be generalized in the framework of the superfield formulation as

$$
\begin{array}{llll}
C \to \pm i\bar{C} & B \to \mp i\mathcal{B} & (\partial \cdot A) \to \pm iE & A_\mu \to A_\mu \\
\bar{C} \to \pm iC & \mathcal{B} \to \mp iB & E \to \pm i(\partial \cdot A) & S_\mu \to S_\mu \\
R_\mu \to -R_\mu & \bar{R}_\mu \to -\bar{R}_\mu & \theta \to -\theta & \bar{\theta} \to -\bar{\theta} \\
\partial_\mu \to \pm i\varepsilon_{\mu\nu}\partial^\nu
\end{array}
\tag{5.3}
$$

under which the superfields transform as

$$
\Phi_\mu(x, \theta, \bar{\theta}) \to \Phi_\mu(x, \theta, \bar{\theta}) \qquad \eta(x, \theta, \bar{\theta}) \to \pm i\bar{\eta}(x, \theta, \bar{\theta}) \qquad \bar{\eta}(x, \theta, \bar{\theta}) \to \pm i\eta(x, \theta, \bar{\theta}).
\tag{5.4}
$$

In the limit: $\theta \to 0, \bar{\theta} \to 0$, we get back transformations (2.9) from (5.3). It is interesting to point out that all the basic properties of the superfields $\eta^2 = \bar{\eta}^2 = 0, \eta\bar{\eta}+\bar{\eta}\eta = 0, [\Phi_\mu, \Phi_\nu] = 0$ remain unchanged under all the discrete symmetry transformations (5.1) and (5.3). Now the Hodge decomposed versions for the 2D vectors $(R_\mu, \bar{R}_\mu, S_\mu)$, consistent with the discrete transformations (5.3), are

$$
R_\mu = \partial_\mu C + \varepsilon_{\mu\nu}\partial^\nu\bar{C} \qquad \bar{R}_\mu = \partial_\mu\bar{C} + \varepsilon_{\mu\nu}\partial^\nu C \qquad S_\mu = \partial_\mu B + \varepsilon_{\mu\nu}\partial^\nu\mathcal{B}
\tag{5.5}
$$

which transform as: $R_\mu \to -R_\mu, \bar{R}_\mu \to -\bar{R}_\mu, S_\mu \to S_\mu$ under the transformations: $C \to \pm i\bar{C}, \bar{C} \to \pm iC, B \to \mp i\mathcal{B}, \mathcal{B} \to \mp iB, \partial_\mu \to \pm i\varepsilon_{\mu\nu}\partial^\nu$. It should be pointed out here that in the derivation of the decompositions (5.2) and (5.5), the continuous symmetry transformations derived in (3.8), (3.11), (4.4) and (4.6) do play an important role.

It is important to note that the form of the Hodge decomposed versions (5.2) differs from the decompositions (5.5) *only* by a sign factor. Thus, the decompositions in (5.2) and (5.5) are *orthogonal* to each other. This shows that the requirement of the consistency of the Hodge decompositions with the discrete symmetries of the theory forces two independent BRST-type nilpotent, continuous and covariant symmetries for the theory. These symmetries are none other than the (anti-)BRST- and the (anti-)co-BRST symmetries we have discussed so far in the previous sections. *It is very interesting to note that the Hodge decomposed versions for the 2D fermionic vectors R_μ and $\bar{R}_\mu$ in (5.2) are also invariant under the discrete symmetry (duality) transformations (2.9).* Thus, they can be called 'self-dual' as they transform to themselves under (2.9).

6. Superfield approach to bosonic symmetry transformation

It is useful for our discussions to calculate the operation of the ordinary Laplacian operator $\Delta = d\delta + \delta d$ on one-form $A = dx^\mu A_\mu$ defined on the 2D flat ordinary Minkowskian spacetime manifold. In fact, it turns out that the one-form ΔA can be expressed as

$$
\Delta(dx^\mu A_\mu) = dx^\mu[\partial_\mu(\partial \cdot A) - \varepsilon_{\mu\nu}\partial^\nu E] \equiv dx^\mu \square A_\mu.
\tag{6.1}
$$

Thus, it is obvious that the operation of Δ on a form does not change the degree of the form. Equation (6.1) is going to be useful for the application of the analogue of the horizontality condition in the context of the operation of the super Laplacian operator on the connection super one-form $\tilde{A}$. This condition can be succinctly expressed as the following equality:

$$
\tilde{\Delta}\tilde{A} = (\tilde{d}\tilde{\delta} + \tilde{\delta}\tilde{d})\tilde{A} = dx^\mu \square A_\mu.
\tag{6.2}
$$

In other words, we expand the lhs of the above equation in terms of the superspace differentials and set all the components of $\tilde{\Delta}\tilde{A}$ that are found to be directed towards the Grassmannian

directions $d\theta$ and $d\bar{\theta}$ equal to zero. In fact, (6.2) can be expressed as

$$\tilde{d}\tilde{\delta}\tilde{A} = dx^\rho J_\rho + d\theta J_\theta + d\bar{\theta} J_{\bar{\theta}}$$
$$\tilde{\delta}\tilde{d}\tilde{A} = dx^\rho H_\rho + d\theta H_\theta + d\bar{\theta} H_{\bar{\theta}}$$

(6.3)

where the explicit expressions for the J's are

$$J_\rho = \partial_\rho(\partial \cdot \Phi) - \varepsilon^{\lambda\bar{\theta}}\partial_\rho\left(\partial_\lambda\bar{\eta} + \varepsilon_{\lambda\mu}\partial_{\bar{\theta}}\Phi^\mu\right) + s^{\theta\theta}\partial_\rho\partial_\theta\eta$$
$$+ s^{\bar{\theta}\bar{\theta}}\partial_\rho\partial_{\bar{\theta}}\bar{\eta} - \varepsilon^{\lambda\theta}\partial_\rho\left(\partial_\lambda\eta + \varepsilon_{\lambda\mu}\partial_\theta\Phi^\mu\right) + s^{\theta\bar{\theta}}\partial_\rho(\partial_\theta\bar{\eta} + \partial_{\bar{\theta}}\eta)$$
$$J_\theta = s^{\bar{\theta}\bar{\theta}}\partial_\theta\partial_{\bar{\theta}}\bar{\eta} + \partial_\theta(\partial \cdot \Phi) - \varepsilon^{\lambda\bar{\theta}}\left(\partial_\theta\partial_\lambda\bar{\eta} + \varepsilon_{\lambda\mu}\partial_\theta\partial_{\bar{\theta}}\Phi^\mu\right)$$
$$J_{\bar{\theta}} = s^{\theta\theta}\partial_{\bar{\theta}}\partial_\theta\eta + \partial_{\bar{\theta}}(\partial \cdot \Phi) - \varepsilon^{\lambda\theta}\left(\partial_{\bar{\theta}}\partial_\lambda\eta + \varepsilon_{\lambda\mu}\partial_{\bar{\theta}}\partial_\theta\Phi^\mu\right)$$

(6.4)

and those of H's are

$$H_\rho = \varepsilon_{\rho\sigma}\partial^\sigma\left(\varepsilon^{\mu\nu}\partial_\mu\Phi_\nu\right) + \varepsilon^{\lambda\bar{\theta}}\varepsilon_{\rho\sigma}\partial^\sigma(\partial_\lambda\eta - \partial_{\bar{\theta}}\Phi_\lambda) - s^{\theta\theta}\varepsilon_{\rho\sigma}\partial^\sigma\partial_\theta\bar{\eta}$$
$$- s^{\bar{\theta}\bar{\theta}}\varepsilon_{\rho\sigma}\partial^\sigma\partial_{\bar{\theta}}\eta + \varepsilon^{\lambda\theta}\varepsilon_{\rho\sigma}\partial^\sigma(\partial_\lambda\bar{\eta} - \partial_\theta\Phi_\lambda) - s^{\theta\bar{\theta}}\varepsilon_{\rho\sigma}\partial^\sigma(\partial_{\bar{\theta}}\bar{\eta} + \partial_\theta\eta)$$
$$H_\theta = s^{\theta\theta}\partial_{\bar{\theta}}\partial_\theta\bar{\eta} - \partial_{\bar{\theta}}\left(\varepsilon^{\mu\nu}\partial_\mu\Phi_\nu\right) - \varepsilon^{\lambda\theta}(\partial_{\bar{\theta}}\partial_\lambda\bar{\eta} - \partial_{\bar{\theta}}\partial_\theta\Phi_\lambda)$$
$$H_{\bar{\theta}} = s^{\bar{\theta}\bar{\theta}}\partial_\theta\partial_{\bar{\theta}}\eta - \partial_\theta\left(\varepsilon^{\mu\nu}\partial_\mu\Phi_\nu\right) - \varepsilon^{\lambda\bar{\theta}}(\partial_\theta\partial_\lambda\eta - \partial_\theta\partial_{\bar{\theta}}\Phi_\lambda).$$

(6.5)

In the computations of (6.4) and (6.5), we have exploited the anti-symmetry and symmetry properties of $\varepsilon^{\mu\theta(\bar{\theta})}$ and $s^{\theta\bar{\theta}}$, respectively. We have also used: $(\partial_\theta)^2 = 0$, $(\partial_{\bar{\theta}})^2 = 0$, etc.

Now we impose the analogue of the horizontality condition. In fact, the condition $d\theta(J_\theta + H_\theta) = 0$ leads to setting the coefficients of $s^{\theta\theta}$, $s^{\bar{\theta}\bar{\theta}}$, $\varepsilon^{\mu\theta}$, $\varepsilon^{\mu\bar{\theta}}$ equal to zero. These are

$$\bar{s} = 0 \qquad \partial_\theta(\partial \cdot \Phi) = \partial_{\bar{\theta}}\left(\varepsilon^{\mu\nu}\partial_\mu\Phi_\nu\right) \qquad \partial_\theta\left(\partial_\mu\bar{\eta} + \varepsilon_{\mu\nu}\partial_{\bar{\theta}}\Phi^\nu\right) = 0 \qquad \partial_{\bar{\theta}}(\partial_\mu\bar{\eta} - \partial_\theta\Phi_\mu) = 0.$$

(6.6)

In the language of the component fields of the expansion (3.2), these conditions lead to

$$\partial \cdot S = 0 \qquad \varepsilon^{\mu\nu}\partial_\mu S_\nu = 0 \qquad \partial \cdot \bar{R} = \varepsilon^{\mu\nu}\partial_\mu R_\nu \qquad S_\mu = \partial_\mu B \qquad S_\mu = \varepsilon_{\mu\nu}\partial^\nu\bar{\mathcal{B}}$$

(6.7)

which imply the results that are listed below

$$\bar{R}_\mu = \varepsilon_{\mu\nu}R^\nu \qquad \Box S_\mu = 0 \qquad S_\mu = \tfrac{1}{2}\left[\partial_\mu B + \varepsilon_{\mu\nu}\partial^\nu\bar{\mathcal{B}}\right] \qquad \partial_\mu B - \varepsilon_{\mu\nu}\partial^\nu\bar{\mathcal{B}} = 0. \quad (6.8)$$

It will be noted that conditions $\partial \cdot S = 0$ and $\varepsilon^{\mu\nu}\partial_\mu S_\nu = 0$ imply $\Box S_\mu = 0$. We also obtain $\Box B = 0, \Box\bar{\mathcal{B}} = 0$. Similarly, the imposition of the constraint $d\bar{\theta}(J_{\bar{\theta}} + H_{\bar{\theta}}) = 0$, leads to

$$s = 0 \qquad \partial_{\bar{\theta}}(\partial \cdot \Phi) = \partial_\theta\left(\varepsilon^{\mu\nu}\partial_\mu\Phi_\nu\right) \qquad \partial_{\bar{\theta}}\left(\partial_\mu\eta + \varepsilon_{\mu\nu}\partial_\theta\Phi^\nu\right) = 0 \qquad \partial_\theta(\partial_\mu\eta - \partial_{\bar{\theta}}\Phi_\mu) = 0.$$

(6.9)

The above equations, *vis-a-vis* expansions in (3.2), lead to

$$\partial \cdot S = 0 \qquad \varepsilon^{\mu\nu}\partial_\mu S_\nu = 0 \qquad \partial \cdot R = \varepsilon^{\mu\nu}\partial_\mu\bar{R}_\nu \qquad S_\mu = -\partial_\mu\bar{B} \qquad S_\mu = -\varepsilon_{\mu\nu}\partial^\nu\mathcal{B}.$$

(6.10)

These equations imply the following results:

$$R_\mu = \varepsilon_{\mu\nu}\bar{R}^\nu \qquad \Box S_\mu = 0 \qquad S_\mu = -\tfrac{1}{2}\left[\partial_\mu\bar{B} + \varepsilon_{\mu\nu}\partial^\nu\mathcal{B}\right] \qquad \partial_\mu\bar{B} - \varepsilon_{\mu\nu}\partial^\nu\mathcal{B} = 0$$

(6.11)

which, in turn, lead to $\Box\bar{B} = 0, \Box\mathcal{B} = 0$. *It will be interesting to point out that the conditions $R_\mu = \varepsilon_{\mu\nu}\bar{R}^\nu$, $\bar{R}_\mu = \varepsilon_{\mu\nu}R^\nu$ are satisfied for both the decompositions (5.2) as well as (5.5).* The final requirement due to the analogue of horizontality condition is

$$dx^\rho(J_\rho + H_\rho) = dx^\rho \Box A_\rho.$$

(6.12)

A close look at the expressions for J_ρ, H_ρ and (6.12) shows that

$$\mathrm{d}x^\rho \left[\partial_\rho (\partial \cdot \Phi) + \varepsilon_{\rho\sigma} \partial^\sigma \left(\varepsilon^{\mu\nu} \partial_\mu \Phi_\nu \right) \right] = \mathrm{d}x^\rho \, \Box \, A_\rho \tag{6.13}$$

where the lhs reduces to $\left(\mathrm{d}x^\rho \, \Box \, \Phi_\rho \right)$. This leads to the following restrictions on R_μ, $\bar{R}_\mu$, S_μ:

$$\Box \, R_\mu = 0 \qquad \Box \, \bar{R}_\mu = 0 \qquad \Box \, S_\mu = 0. \tag{6.14}$$

Hereafter, in the rest of computations, we set $s(x) = 0$, $\bar{s}(x) = 0$ in the expansion (3.2) (cf (6.6), (6.9)). Now setting the coefficients of $(\mathrm{d}x^\rho s^{\theta\theta})$ and $(\mathrm{d}x^\rho s^{\theta\bar{\theta}})$ equal to zero leads to

$$\partial_\theta \left(\partial_\rho \eta - \varepsilon_{\rho\sigma} \partial^\sigma \bar{\eta} \right) = 0 \qquad \partial_{\bar{\theta}} \left(\partial_\rho \bar{\eta} - \varepsilon_{\rho\sigma} \partial^\sigma \eta \right) = 0. \tag{6.15}$$

The above equations produce the following two restrictions:

$$\partial_\rho B - \varepsilon_{\rho\sigma} \partial^\sigma \mathcal{B} = 0 \qquad \partial_\rho \bar{B} - \varepsilon_{\rho\sigma} \partial^\sigma \bar{\mathcal{B}} = 0. \tag{6.16}$$

It will be noted that these restrictions automatically satisfy the conditions that emerge due to setting the coefficient of $(\mathrm{d}x^\rho s^{\theta\bar{\theta}})$ equal to zero. A comparison between (6.8) and (6.11) on the one hand and (6.16) on the other hand, produces the conditions: $B = \bar{B}, \mathcal{B} = \bar{\mathcal{B}}$, $\partial_\rho B - \varepsilon_{\rho\sigma} \partial^\sigma \mathcal{B} = 0$. However, with these equalities, it is clear that equations (6.8) and (6.11) lead to: $S_\mu = 0$, $\partial_\rho B + \varepsilon_{\rho\sigma} \partial^\sigma \mathcal{B} = 0$. Thus, we have got the orthogonal combinations of derivatives on B and $\mathcal{B}$ that are equal to zero. Ultimately, these imply that we can choose: $B = \bar{B} = \mathcal{B} = \bar{\mathcal{B}} = 0$. At this stage, it is worthwhile to emphasize that, as the operator equations, these conditions are *not* unexpected. In fact, it has been shown that the choice of the physical state as the harmonic state in the Hodge decomposition allows one to require that: $Q_{a(b)}|\text{phys}\rangle = 0$, $Q_{a(d)}|\text{phys}\rangle = 0$ which imply that $B|\text{phys}\rangle = 0$, $\mathcal{B}|\text{phys}\rangle = 0$ (cf (7.1)) [1, 2, 6]. There is one more subtle point that has to be stressed here. Even though, $S_\mu = 0$ in terms of B and $\mathcal{B}$, there is another choice of S_μ that remains invariant under the discrete symmetry transformations (2.9). For instance, $S_\mu = -\varepsilon_{\mu\nu} \Box A^\nu$ remains invariant under the duality transformations (2.9) because $A_\mu \to A_\mu$ and $\Box \to \Box$ under $\partial_\mu \to \pm i \varepsilon_{\mu\nu} \partial^\nu$. Furthermore, the conditions $\partial \cdot S = 0$ and $\varepsilon^{\mu\nu} \partial_\mu S_\nu = 0$ are automatically satisfied because $\Box(\partial \cdot A) = 0$ and $\Box E = 0$ due to the equations of motion (with $B = -(\partial \cdot A)$, $\mathcal{B} = E$). Thus, we shall persist with S_μ in our further discussions.

For the rest of our computations, we shall set $s(x) = \bar{s}(x) = 0$ and $B(x) = \bar{B}(x) = \mathcal{B}(x) = \bar{\mathcal{B}}(x) = 0$ in our superfield expansion (3.2). With these insertions in (3.2), it is obvious now that setting of the coefficients of $(\mathrm{d}x^\rho \varepsilon^{\lambda\theta})$ and $(\mathrm{d}x^\rho \varepsilon^{\lambda\bar{\theta}})$ equal to zero, leads to

$$\begin{aligned} \varepsilon_{\rho\sigma} \partial^\sigma (\partial_\lambda \bar{\eta} - \partial_\theta \Phi_\lambda) - \partial_\rho \left(\partial_\lambda \eta + \varepsilon_{\lambda\mu} \partial_\theta \Phi^\mu \right) &= 0 \\ \varepsilon_{\rho\sigma} \partial^\sigma (\partial_\lambda \eta - \partial_{\bar{\theta}} \Phi_\lambda) - \partial_\rho \left(\partial_\lambda \bar{\eta} + \varepsilon_{\lambda\mu} \partial_{\bar{\theta}} \Phi^\mu \right) &= 0. \end{aligned} \tag{6.17}$$

In terms of the component fields of (3.2), the above equations produce the following conditions:

$$\begin{aligned} \partial_\lambda \left(\varepsilon_{\rho\sigma} \partial^\sigma \bar{C} - \partial_\rho C \right) &= \partial_\rho R_\lambda + \varepsilon_{\rho\sigma} \partial^\sigma \bar{R}_\lambda \\ \partial_\lambda \left(\varepsilon_{\rho\sigma} \partial^\sigma C - \partial_\rho \bar{C} \right) &= \partial_\rho \bar{R}_\lambda + \varepsilon_{\rho\sigma} \partial^\sigma R_\lambda \\ \varepsilon_{\rho\sigma} \partial^\sigma S_\lambda + \partial_\rho \left(\varepsilon_{\lambda\mu} S^\mu \right) &= 0 \end{aligned} \tag{6.18}$$

where $R_\mu = \varepsilon_{\mu\nu} \bar{R}^\nu$, $\bar{R}_\mu = \varepsilon_{\mu\nu} R^\nu$. The last equation, contracted with ∂_λ, is automatically satisfied due to $(\partial \cdot S) = 0$ and $\varepsilon^{\mu\nu} \partial_\mu S_\nu = 0$. Moreover, the top equation can be re-expressed, in terms of the component fields, as

$$-\partial_\lambda \left(\varepsilon_{\rho\sigma} \partial^\sigma C - \partial_\rho \bar{C} \right) = \partial_\rho \bar{R}_\lambda + \varepsilon_{\rho\sigma} \partial^\sigma R_\lambda. \tag{6.19}$$

Comparison of the central equation of (6.18) with the above equation, yields

$$\partial_\lambda \left(\varepsilon_{\rho\sigma} \partial^\sigma C - \partial_\rho \bar{C} \right) = 0 \qquad \partial_\rho \bar{R}_\lambda + \varepsilon_{\rho\sigma} \partial^\sigma R_\lambda = 0. \tag{6.20}$$

One of the very interesting solutions for the above conditions is $\bar{R}_\rho = \partial_\rho \bar{C} - \varepsilon_{\rho\sigma} \partial^\sigma C = 0$ which is equivalent to setting the Hodge decomposed version of equation (5.2) equal to zero. This, in turn, implies that the other 2D fermionic vector $R_\rho = \partial_\rho C - \varepsilon_{\rho\sigma} \partial^\sigma \bar{C}$ is also equal to zero. Thus, ultimately, we have obtained the following solution from the analogue of the horizontality condition wrt the super Laplacian operator $\tilde{\Delta}$

$$R_\mu(x) = \bar{R}_\mu(x) = 0 \qquad B(x) = \bar{B}(x) = \mathcal{B}(x) = \bar{\mathcal{B}}(x) = 0$$
$$S_\mu(x) = -\varepsilon_{\mu\nu} \square A^\nu(x) \equiv (\tilde{s}_w A_\mu(x)) \qquad s(x) = \bar{s}(x) = 0. \tag{6.21}$$

The superfield expansion (3.2), with the above inputs, has the appearance

$$\Phi_\mu(x, \theta, \bar{\theta}) = A_\mu(x) + \mathrm{i}\theta\bar{\theta}(\tilde{s}_w A_\mu(x)) \qquad \eta(x, \theta, \bar{\theta}) = C(x) \qquad \bar{\eta}(x, \theta, \bar{\theta}) = \bar{C}(x).$$
$$\tag{6.22}$$

This shows that the ghost fields $C(x)$ and $\bar{C}(x)$ do not transform at all under $\tilde{s}_w$ and gauge field $A_\mu(x)$ transforms to its own equation of motion along the bosonic $(\theta\bar{\theta})$ direction of the expansion. This establishes the fact that symmetry transformations $\tilde{s}_w C = 0, \tilde{s}_w \bar{C} = 0, \tilde{s}_w A_\mu = -\varepsilon_{\mu\nu} \square A^\nu$, which are the analogues of (2.7) (with $\mathcal{B} = E$, $B = -(\partial \cdot A)$), are generated by a conserved bosonic charge Q_w (cf (7.1)) which owes its origin to the super Laplacian operator $\tilde{\Delta}$. Geometrically, the operator Q_w turns out to be the translation generator along a bosonic direction that is equivalent to a couple of successive translations along the θ and $\bar{\theta}$ directions (i.e. $\mathrm{Lim}_{\theta,\bar{\theta}\to 0} \frac{\partial}{\partial\theta} \frac{\partial}{\partial\bar{\theta}} \Phi_\mu(x, \theta, \bar{\theta}) = [A_\mu, Q_w]$).

7. Conclusions

In the present investigation, we have exploited the mathematical power of super de Rham cohomology operators $(\tilde{d}, \tilde{\delta}, \tilde{\Delta})$ of differential geometry and a generalized version of the so-called horizontality condition [13–18] for the geometrical interpretation of the local Noether conserved charges $(Q_{(a)b}, Q_{(a)d}, Q_w)$ as the generators of translations along the Grassmannian (odd) and bosonic (even) directions of a compact (2+2)-dimensional supermanifold. The explicit *local* expressions for all these conserved Noether charges, together with the ghost charge Q_g, are [1, 2, 6]

$$Q_b = \int \mathrm{d}x[B\dot{C} - \dot{B}C] \qquad Q_{ab} = \int \mathrm{d}x[B\dot{\bar{C}} - \dot{B}\bar{C}]$$
$$Q_d = \int \mathrm{d}x[\mathcal{B}\dot{\bar{C}} - \dot{\mathcal{B}}\bar{C}] \qquad Q_{ad} = \int \mathrm{d}x[\mathcal{B}\dot{C} - \dot{\mathcal{B}}C] \tag{7.1}$$
$$Q_w = \int \mathrm{d}x[B\dot{\mathcal{B}} - \dot{B}\mathcal{B}] \qquad Q_g = -\mathrm{i}\int \mathrm{d}x[C\dot{\bar{C}} + \bar{C}\dot{C}]$$

which obey the following algebra [1, 2, 6]:

$$Q_b^2 = Q_d^2 = Q_{ab}^2 = Q_{ad}^2 = 0 \qquad \{Q_b, Q_{ab}\} = \{Q_d Q_{ad}\} = 0$$
$$\mathrm{i}[Q_g, Q_{b(ad)}] = +Q_{b(ad)} \qquad \mathrm{i}[Q_g, Q_{d(ab)}] = -Q_{d(ab)} \tag{7.2}$$
$$Q_w = \{Q_b, Q_d\} = \{Q_{ad}, Q_{ab}\} \qquad [Q_w, Q_k] = 0 \qquad k = a, ab, d, ad, g$$

if we exploit the canonical (anti-)commutation relations for the Lagrangian density (2.2). The above algebra is reminiscent of the algebra obeyed by the de Rham cohomology operators of differential geometry (see, e.g., section 1) and it remains form-invariant under the discrete duality transformations (2.9). The degree of a given differential form is equivalent to the ghost number of a quantum state in the BRST formalism as (1.1) and (7.2) imply

$$iQ_g Q_{b(ad)}|\Psi\rangle_n = (n+1)Q_{b(ad)}|\Psi\rangle_n$$
$$iQ_g Q_{d(ab)}|\Psi\rangle_n = (n-1)Q_{d(ab)}|\Psi\rangle_n \qquad (7.3)$$
$$iQ_g Q_w|\Psi\rangle_n = nQ_w|\Psi\rangle_n.$$

The above equations re-establish the identification of conserved local Noether charges with the de Rham cohomology operators of differential geometry because as operators $d(\delta)$ increase (reduce) the degree of a form by one so do the local charges $Q_{b(ad)}$ $(Q_{d(ab)})$ to a state with a certain ghost number. Thus, one has the following mappings: $d \Leftrightarrow (Q_b, Q_{ad}), \delta \Leftrightarrow (Q_d, Q_{ab}), \Delta \Leftrightarrow Q_w = \{Q_{(a)b}, Q_{(a)d}\}$. With these identifications, the HDT and the BRST cohomology can be defined comprehensively in the quantum Hilbert space of states [1, 2, 6, 11]. The existence of some new discrete symmetries in the superfield formulation leads to some key insights which enable us to obtain the Hodge decomposition for the 2D odd vectors of the superfield expansion (3.2). The interplay between discrete and continuous symmetries of the theory (together with the idea of the HDT) provides the origin for the existence of (anti-)BRST and (anti-)co-BRST symmetries. The requirement of the self-duality invariance of the theory under (2.9) provides a logical reason for the choice of the Hodge decomposed version (5.2) over its counterpart (5.5) in the solution (6.20). In fact, this solution finally leads to the derivation of the bosonic symmetry where $R_\mu = \bar{R}_\mu = 0$. The self-duality invariance also provides a very convincing guess for the correct Hodge decomposed version of 2D bosonic vectors: $S_\mu = \partial_\mu B + \varepsilon_{\mu\nu}\partial^\nu \mathcal{B}$ and $A_\mu = \partial_\mu \lambda + \varepsilon_{\mu\nu}\partial^\nu \kappa$ where λ and κ are some scalar fields (see, e.g., [2] for details).

In our earlier works [6, 1, 2], it has been demonstrated that 2D free Abelian $U(1)$ gauge theory in the flat Minkowski spacetime is a new type of topological field theory (TFT) [23] as the BRST and dual BRST symmetries gauge out the propagating degrees of freedom of the 2D photon. This new theory captures together some of the key features of Witten- as well as Schwarz type TFTs [24, 25]. In fact, the form of the quantum action for this theory bears the appearance of a Witten type theory but the symmetries are local gauge type which very much resemble symmetries of the Schwarz type TFT. The topological nature of this theory has been shown to be encoded in the vanishing of the Laplacian operator $Q_w \to 0$ when the on-shell condition for a photon, i.e. $(\partial_\mu B + \varepsilon_{\mu\nu}\partial^\nu \mathcal{B} = 0)$, is utilized [1, 2, 6]. In fact, under this condition, all the transformations in (2.7) become trivially zero. In the framework of superfield formulation, this fact is encoded in the vanishing of the 2D fermionic vectors R_μ and $\bar{R}_\mu$ and the bosonic 2D vector S_μ turns out to be equal to $-\varepsilon_{\mu\nu}\Box A^\nu$ only. Thus, on the on-shell (i.e. $\Box A_\mu = 0$), even S_μ turns out to be zero. The topological nature of the 2D free $U(1)$ gauge theory is captured in the horizontality condition of the superfield formulation. In fact, it can be seen that the restriction $\tilde{\Delta}\tilde{A} = \Delta A (\equiv dx^\rho \Box A_\rho)$ vanishes on the on-shell ($\Box A_\mu = 0$) of the 2D photon which, ultimately, implies $Q_w \to 0$. In a recent paper [26], a very direct, simple and straightforward method has been adopted to capture some of the topological features of our present theory where the explicit presence of the superfields $\Phi_\mu(x, \theta, \bar{\theta}), \eta(x, \theta, \bar{\theta})$ and $\bar{\eta}(x, \theta, \bar{\theta})$ has been invoked.

It is important to point out that, in the superfield formulation on the (2+2)-dimensional supermanifold, one can define forms that have greater degree than the spacetime dimension of the supermanifold. However, the generalized versions of the horizontality condition w.r.t. all the super cohomological operators (which we have exploited in our present discussion) turn out to be quite handy to resolve this anomaly. It would be nice to take an explicit example of a supersymmetric theory and work out all the details of the HDT for this theory in the quantum Hilbert space of states. At the moment, we are looking into this problem and our results will be reported in our future publications.

An interesting generalization of our present superfield formulation for the free 2D Abelian gauge theory to self-interacting 2D non-Abelian gauge theory has already been studied in [27].

It would be a very interesting endeavour to look for the field theoretical models for the Hodge theory where there is an interaction between matter and gauge fields. Some steps in this direction have already been taken [3, 4] for the interacting 2D Abelian gauge theory. It has been shown in these works that, as (anti-)BRST symmetries are connected with the local gauge symmetries on gauge and Dirac fields, similarly the (anti-)co-BRST symmetries on the gauge fields are connected with the analogue of chiral symmetries on the Dirac fields. The local Noether charges, corresponding to these symmetries, turn out to be the analogues of cohomological operators d and δ. We firmly believe that a thorough study of this interacting theory with the Wess–Zumino term would shed light on the consistency and unitarity of the 2D anomalous gauge theory (see, e.g., [28, 29] and references therein) in the framework of BRST cohomology and HDT. It would also be interesting to generalize our results to the free 4D two-form Abelian gauge theory where the existence of analogues of the local conserved charges $Q_{(a)b}$, $Q_{(a)d}$, Q_w has been established. These are some of the issues that are under investigation [30].

Acknowledgments

References

[1] Malik R P 2000 *J. Phys. A: Math Gen* **33** 2437 (hep-th/9902146)
[2] Malik R P 2000 *Int. J. Mod. Phys*. A **15** 1685 (hep-th/9808040)
[3] Malik R P 2000 *Mod. Phys. Lett*. A **15** 2079 (hep-th/0003128)
[4] Malik R P 2001 *Mod. Phys. Lett*. A **16** 477 (hep-th/9711056)
[5] Malik R P 1999 *Mod. Phys. Lett*. A **14** 1937 (hep-th/9903121)
[6] Malik R P 2001 *J. Phys. A: Math Gen* **34** 4167 (hep-th/0012085)
[7] Eguchi T, Gilkey P B and Hanson A J 1980 *Phys. Rep*. **66** 213
[8] Mukhi S and Mukunda N 1990 *Introduction to Topology, Differential Geometry and Group Theory for Physicists* (New Delhi: Wiley)
[9] Nishijima K 1988 *Prog. Theor. Phys*. **80** 897
    van Holten J W 1990 *Phys. Rev. Lett*. **64** 2867
    Aratyn H 1990 *J. Math. Phys*. **31** 1240
[10] Fülöp G and Marnelius R 1995 *Nucl. Phys*. B **456** 442
    Kalau W and van Holten J W 1991 *Nucl. Phys*. B **261** 233
[11] Harikumar E, Malik R P and Sivakumar M 2000 *J. Phys. A: Math Gen* **33** 7149 (hep-th/0004145)
[12] Malik R P 2001 *Phys. Lett*. B **521** 409 (hep-th/0108105)
[13] Thierry-Mieg J 1980 *J. Math. Phys*. **21** 2834
    Thierry-Mieg J 1980 *Nuovo Cimento* A **56** 396
[14] Quiros M, De Urries F J, Hoyos J, Mazou M L and Rodrigues E 1981 *J. Math. Phys*. **22** 767
[15] Bonora L and Tonin M 1981 *Phys. Lett*. B **98** 48
    Bonora L, Pasti P and Tonin M 1981 *Nuovo Cimento* A **63** 353
[16] Baulieu L and Thierry-Mieg J 1982 *Nucl. Phys*. B **197** 477
    Baulieu L and Thierry-Mieg J 1982 *Nucl. Phys*. B **228** 259
    Alvarez-Gaumé L and Baulieu L 1983 *Nucl. Phys*. B **212** 255
[17] Hwang D S and Lee C-Y 1997 *J. Math. Phys*. **38** 30
[18] Nakanishi N and Ojima I 1990 *Covariant Operator Formalism of Gauge Theories and Quantum Gravity* (Singapore: World Scientific)
[19] Nishijima K 1986 *Progress in Quantum Field Theory* ed H Ezawa and S Kamefuchi (Amsterdam: North-Holland)
[20] Henneaux M and Teitelboim C 1992 *Quantization of Gauge Systems* (Princeton, NJ: Princeton University Press)

[21] Weinberg S 1996 *The Quantum Theory of Fields: Modern Applications* vol 2 (Cambridge: Cambridge University Press)
[22] Wess J and Bagger J 1991 *Supersymmetry and Supergravity* 2nd edn (Princeton, NJ: Princeton University Press)
[23] Birmingham D, Blau M, Rakowski M and Thompson G 1991 *Phys. Rep.* **209** 129
[24] Witten E 1989 *Commun. Math. Phys.* **121** 351
[25] Schwarz A S 1978 *Lett. Math. Phys.* **2** 217
[26] Malik R P 2001 Topological aspects of Abelian gauge theory in superfield formalism *Preprint* hep-th/0112260
[27] Malik R P 2002 *Mod. Phys. Lett.* A **17** 185 (hep-th/0111253)
[28] Jackiw R and Rajaraman R 1985 *Phys. Rev. Lett.* **54** 1219
[29] Malik R P 1988 *Phys. Lett.* B **212** 445
[30] Malik R P 2002 in preparation